Short Papers

Exact Analysis of Coupled Nonuniform Transmission Lines with Exponential Power Law Characteristic Impedance

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***Abstract*—A new method for frequency domain and transient analysis of the multiple coupled lossless nonuniform transmission lines with general exponential power law characteristic impedance will be presented in this paper. First, the analytical solution in frequency domain (as $ABCD$ and S-parameters matrices) is obtained using the Frobenius method, and then a simple fast Fourier transform algorithm is used to find the time-domain response of the lines.**

***Index Terms*—Nonuniform transmission lines, transient response.**

I. Introduction

Nonuniform transmission lines (NTLs) have been widely used in several microwave applications. Analysis and simulation of single and coupled NTLs in both time and frequency domains have been presented in several papers using different methods [1]–[11]. Unfortunately, the general solution for arbitrary NTLs does not exist analytically, unless for some special kinds of transmission lines. For example, [1] presents an exact solution for single ideal linear varied NTL. This solution was extended for the case of two coupled transmission lines in [2]. Single Hermite transmission line was considered in [3]. The authors of [4] consider a single lossless NTL with power law characteristic impedance in time domain and the authors of [5] present a solution for the case of two NTLs with the same power law characteristic impedance using Bessel functions with fractional orders. The same problem was also considered in [6].

Also, several methods have been presented to analyze different kinds of microstrip coupled NTLs. For example, [7] investigated the tapered multiple microstrip lines using the spatial iteration–perturbation approach technique, and [8] analyzed the problem using a time-domain scattering-parameter formulation incorporated with the closed-form expressions of voltage variables for divided short uniform lossless lines. The method of step-lines approximations [9] and a cascaded network chain [10] was also used to solve the same problem.

In this paper, a simple method for analyzing multiple coupled lossless NTLs with exponential power law characteristic impedance (EPLCI) is presented. The geometry under considerations includes M (arbitrary number) of coupled NTLs with the same EPLCI profile. The method of the solution is based on the modal decomposition method [11]. Using the modal decomposition method, the system of coupled partial differential equations are decomposed to a number of uncoupled ordinary wave equations, which are then solved analytically in the frequency domain using the Frobenius method. Using this solution, an exact $ABCD$ matrix and S-parameter matrix are obtained. Finally, a fast Fourier transform (FFT) algorithm is used to find the transient response.

Manuscript received July 1, 1999.

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Publisher Item Identifier S 0018-9480(01)00026-6.

II. Analytical Frequency-Domain Solution

Consider a system of M-coupled NTLs with the same EPLCI profile. The lengths of the lines are taken to be equal to d. The partial differential equations describing this structure are given by

$$\frac{\partial [V(z)]}{\partial z} + j\omega [L(z)][I(z)] = 0 \tag{1}$$

$$\frac{\partial [I(z)]}{\partial z} + j\omega [C(z)][V(z)] = 0 \tag{2}$$

where $[V(z)]$ and $[I(z)]$ are $M \times 1$ voltage and current vectors defined as

$$[V(z)]^T = [V_1(z) \quad V_2(z) \quad \cdots \quad V_M(z)] \tag{3}$$

in which the superscript T indicates the transpose of the matrix (vector), and $V_k(z)$ represents the voltage along the kth line. The same definition can be made for $[I(z)]$. The capacitance and inductance matrices for this system are defined by

$$[C(z)] = \exp(-qz)(1+pz)^{-2N}[C]_o \tag{4}$$

$$[L(z)] = \exp(qz)(1+pz)^{2N}[L]_o \tag{5}$$

in which z is the position along the lines, and N (an integer), p, and q are arbitrary constants. $[C]_o$ and $[L]_o$ are constant $M \times M$ matrices. Using the *modal decomposition* method [11], one may decouple (1) and (2) by simultaneously diagonalizing $[L]_o$ and $[C]_o$ matrices. The modal variables are defined by

$$[V^m(z)] = [Q_v][V(z)] \tag{6}$$

$$[I^m(z)] = [Q_i][I(z)]. \tag{7}$$

A simple method to obtain the constant *transfer matrices* $[Q_v]$ and $[Q_i]$ for known $[C]_o$ and $[L]_o$ is given in [11]. Substituting (6) and (7) in (1) and (2) leads to the uncoupled set of partial differential equations for the modal variables as

$$\frac{\partial [V^m(z)]}{\partial z} + \exp(qz)(1+pz)^{2n}[L^m][I^m(z)] = 0 \tag{8}$$

$$\frac{\partial [I^m(z)]}{\partial z} + \exp(-qz)(1+pz)^{-2N}[C^m][V^m(z)] = 0. \tag{9}$$

The propagation constant for the kth mode is defined as

$$\beta_{ok}^m = \omega\sqrt{L_k^m C_k^m} \tag{10}$$

where L_k^m and C_k^m are elements of the diagonal matrices $[L^m]$ and $[C^m]$, respectively, defined as

$$[L^m] = [Q_v][L]_o[Q_i]^{-1} \tag{11}$$

$$[C^m] = [Q_i][C]_o[Q_v]^{-1}. \tag{12}$$

Thus, by using proper transfer matrices $[Q_v]$ and $[Q_i]$, the coupled equations (1) and (2) are now decoupled to M uncoupled wave equations in terms of modal variables. The differential equation describing the voltage of the kth mode in this structure is

$$\frac{\partial^2 V_k^m(y)}{\partial y^2} + \left(q_o + \frac{2N}{y}\right)\frac{\partial V_k^m(y)}{\partial y} + \left(\frac{\beta_{ok}^m}{p}\right)^2 V_k^m(y) = 0. \quad (13)$$

In which $k = 1, 2, \ldots, M$, $q_o = (q/p)$, and $y = 1 + pz$. Also, the differential equation describing the current of the kth mode can be derived simply.

In the Frobenius method, we are looking for a solution as

$$V_k^m(y) = \sum_{n=0}^{\infty} a_{kn}^m(s) y^{s+n} \quad (14)$$

in which s and $a_{kn}^m(s)$ are constants to be determined. Substituting (14) in (13) yields

$$\sum_{n=0}^{\infty}(s+n)(s+n-1)a_{kn}^m(s)y^{s+n-2} - (q_o y + 2N)\sum_{n=0}^{\infty} \cdot (s+n)a_{kn}^m(s)y^{s+n-2} + \left(\frac{\beta_{ok}^m}{p}\right)^2 \sum_{n=0}^{\infty} a_{kn}^m(s)y^{s+n} = 0. \quad (15)$$

This leads to the following identities:

$$s(s-1-2N)a_{k0}(s)^m = 0 \quad (16)$$

$$a_{k1}^m(s) = \frac{q_o s}{(s+1)(s-2N)}\, a_{k0}^m(s) \quad (17)$$

$$a_{k(n+2)}^m(s) = \frac{q_o(s+n+1)a_{k(n+1)}^m(s) - \left(\frac{\beta_{ok}^m}{p}\right)^2 a_{kn}^m(s)}{(n+s+2)(-2N+n+s+1)}. \quad (18)$$

There are two solutions for (16), i.e., for $s = 0$ and $s = 2N+1$. Convergence of (18) can be easily shown. Therefore, using the recurrence relation (18) with (17) [$a_{k0}^m(s)$ is arbitrary chosen to be equal to one] and applying proper boundary conditions, the voltage expression can be derived exactly for the kth mode as

$$V_k^m(z) = A_0 \sum_{n=0}^{\infty} a_{kn}^m(0)(1+pz)^n + A_1 \sum_{n=0}^{\infty} a_{kn}^m(2N+1) \cdot (1+pz)^{n+2N+1} \quad (19)$$

$a_{kn}^m(0)$ and $a_{kn}^m(2N+1)$ are obtained from (18) for $s = 0$ and $s = 2N+1$, respectively. A_0 and A_1 are determined from boundary conditions. The same procedure can be followed to obtain the solution

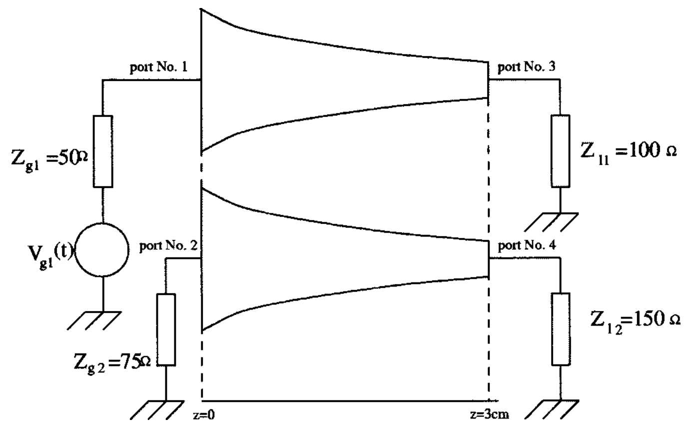

Fig. 1. Two coupled transmission lines with EPLCI.

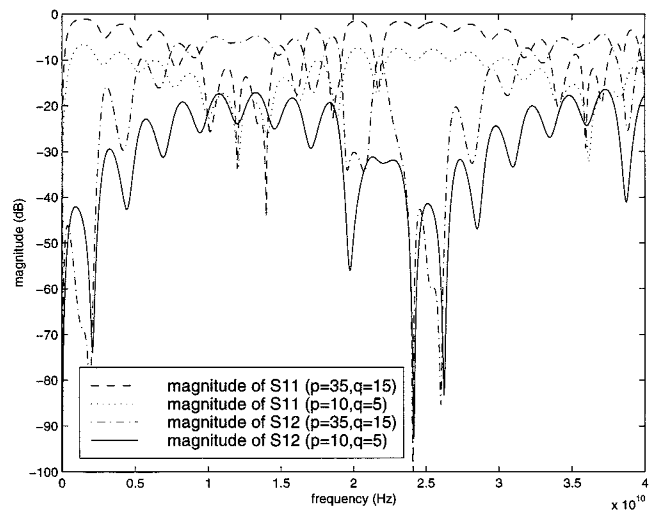

Fig. 2. Comparison of the magnitudes of S_{11} and S_{12} for $p = 35m^{-1}$, $q = 15m^{-1}$, and $p = 10m^{-1}$, $q = 5m^{-1}$.

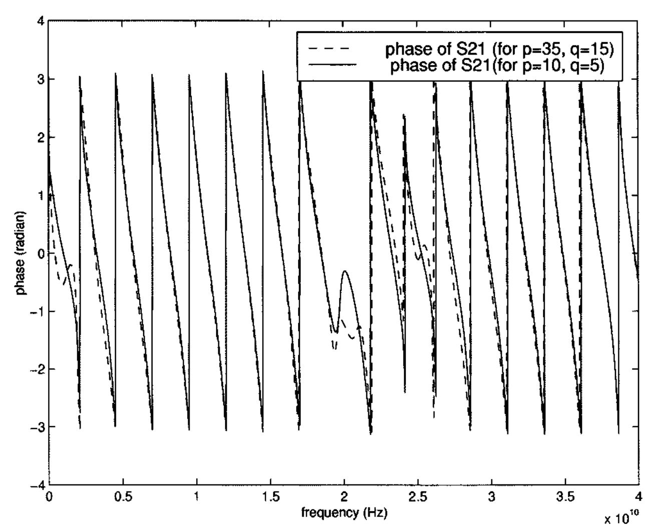

Fig. 3. Comparison of the phase of S_{21} for $p = 35m^{-1}$, $q = 15m^{-1}$, and $p = 10m^{-1}$, $q = 5m^{-1}$.

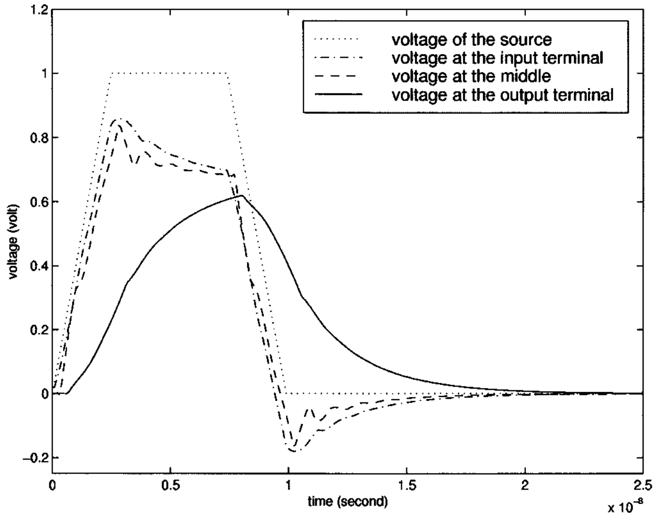

Fig. 4. Input voltage source and voltages at various points of the excited line for $p = 35m^{-1}$, $q = 15m^{-1}$.

for the current kth mode. Therefore, the total $ABCD$ matrix for the modal lines is obtained as

$$\begin{bmatrix} V_1^m(d) \\ I_1^m(d) \\ V_2^m(d) \\ I_2^m(d) \\ \vdots \\ V_{(M-1)}^m(d) \\ I_{(M-1)}^m(d) \\ V_M^m(d) \\ I_M^m(d) \end{bmatrix} = \begin{bmatrix} [T_1^m(d) & 0 & \cdots & 0 & 0 \\ 0 & [T_2^m(d)] & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & [T_{M-1}^m(d)] & 0 \\ 0 & 0 & \cdots & 0 & [T_M^m(d)] \end{bmatrix} \cdot \begin{bmatrix} V_1^m(0) \\ I_1^m(0) \\ V_2^m(0) \\ I_2^m(0) \\ \vdots \\ V_{(M-1)}^m(0) \\ I_{(M-1)}^m(0) \\ V_M^m(0) \\ I_M^m(0) \end{bmatrix}. \quad (20)$$

Finally, the $ABCD$ matrix $([T])$ of the main lines can be derived using (6) and (7) in (20). The method presented here is used to obtain the S-parameter's matrix from the $ABCD$ matrix.

III. Examples and Results

Consider a set of two coupled lines each with length $d = 3$ cm (see Fig. 1). Here, we assume

$$[L]_o = \begin{bmatrix} 356.8 & 71.36 \\ 71.36 & 356.8 \end{bmatrix} \mu\text{H/m}$$
$$[C]_o = \begin{bmatrix} 124.9 & -12.4 \\ -12.4 & 124.13 \end{bmatrix} \text{pF/m.}$$

Due to the symmetrical property of the system, the S-parameters matrix has only eight independent elements. The magnitude and phase of some of the independent elements of the S-parameters matrix are compared for two cases $p = 35m^{-1}$, $q = 15m^{-1}$, and $p = 10m^{-1}$, $q = 5m^{-1}$, both for $N = 1$, in Figs. 2–3.

The time-domain response in this structure can be obtained using the FFT algorithm. The voltage waveforms at the input terminals of the lines for the case $p = 35$ m^{-1}, $q = 15$ m^{-1}, and $N = 1$ is shown in Fig. 4.

IV. Conclusions

An exact $ABCD$ and S-parameter's matrices for the system of coupled lossless NTLs with general EPLCI have been presented in this paper. The transient voltage and current at various points of the lines are derived using the FFT algorithm. This approach can be used for several microwave applications such as filter and interconnect matching network design. The presented method can be generalized to consider the effect of the external electromagnetic fields, which are the subject of the author's future work.

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